Bianchi type-IX cosmological model with anisotropic dark energy

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Abstract— The exact solutions of the Einstein field equations for dark energy in Bianchi type-IX metric under the assumption on the anisotropy of the fluid are obtained for exponential and power law volumetric expansions. The isotropy of the fluid, space and expansion are examined.

Keywords: Bianchi type-IX space-time, dark energy, anisotropic fluid, isotropization

1 Introduction

In 21'st century, the greatest problem arises that, 'How to understand the nature and origin of dark matter and dark energy.' The presence of dark energy opposes the self-attraction of matter and causes the expansion of universe to accelerate; this is indicated in the survey of cosmological distinct type Ia Supernovae [15,18]. This was confirmed in the further observations [26, 19, 9]. Also our universe is dominated by dark energy (DE) with $\cong 3/4$ of the critical density is strongly indicated from the observations [8,11,12]. Now the problem lies in detecting an exotic type of unknown repulsive force, termed as DE which is strongly responsible for an accelerating phase of universe. The detection of DE would be a new clue to an old puzzle: the gravitational effect of the zero point energies of particles and fields [29,30]. In the late 1990's, the term DE was first introduced after the brightness of distinct supernovas exploding stars was studied. Those observations supposed that, the universe is mainly filled with three components: 4% for baryonic matter, 23% for nonbaryonic dark matter and 73% so called DE [21]. A matter without pressure and DE is an exotic energy with negative pressure called as dark matter. The existence of DE fluids come from the observations of the accelerated expansion of the universe. The isotropic pressure of the cosmological models give the best fitting of the observations.

Many relativists have taken a keen interest in studying Bianchi type-IX universes because familiar solutions like the Robertson-Walker universe with positive curvature, the de-sitter universe , the Taub-Nut solutions etc. are of Bianchi type-IX space times. Bali *et al.* [6, 7] studied Bianchi type-IX string cosmological models with bulk viscous fluid distribution in general relativity. Pradhan *et al.* [16] have investigated some homogeneous Bianchi type-IX viscous fluid cosmological models with a varying Λ. Tyagi *et al.* [27] have established Bianchi type-IX string cosmological models for perfect fluid distribution in general relativity. Reddy *et al.* [17] have presented Bianchi type-IX cosmic strings in a scalar tensor theory of gravitation.

The anisotropy of the DE within the framework of Bianchi type space times is found to be useful in generating arbitrary ellipsoidality to the universe and to fine tune the observed CMBR anisotropies. Koivisto and Mota [13] have investigated cosmological models with anisotropic EoS. Akarsu *et al.*[5] have investigated Bianchi type-I anisotropic dark energy model with constant deceleration parameter. Yadav *et al.* [28] have studied Bianchi type-III anisotropic DE models with constant deceleration parameter. Suresh kumar *et al.* [23] have studied Bianchi type-V model of accelerating universe with DE. Suresh kumar [24] have investigated Bianchi type-II model in the presence of perfect fluid and anisotropic DE. Recently Adhav *et al.* [1, 2, 3, 4] have studied Kantowski-Sachs, Kaluza-Klein LRS Bianchi Type-II and Bianchi Type-VI$_0$ cosmological models with anisotropic DE. To have a general description of anisotropic DE component, we consider a phenomenological parameterization of DE in terms of its equation of state (ω) and two skewness parameters (δ, γ).

In this paper, first the general form of the anisotropy parameter of the expansion for Bianchi type-IX metric is obtained. Here the skewness parameters δ and γ are equal which are the deviations from ω on y and z axes respectively. The exact solutions of the Einstein's field equations have been obtained by reducing anisotropic parameter of the expansion to a simple form for volumetric exponential expansion and power law expansion. Some features of the evolution of the metric and the dynamics of the anisotropic DE fluid have been examined.

2. FIELD EQUATIONS :

Bianchi type-IX metric is considered in the form,

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2\sin^2 y + a^2\cos^2 y)dz^2 - 2a^2\cos y dx dz \tag{1}$$

where a, b are scale factors and are functions of cosmic time t.

The Einstein's field equations in natural units ($8\pi G = 1$ and $C = 1$) are given by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}\,, \tag{2}$$

where $u^i = (1,0,0,0)$ is the four velocity vector and $g_{ij}u^iu^j = -1$; R_{ij} is a Ricci tensor, R is a Ricci scalar, T_{ij} is an energy-momentum tensor.

The energy-momentum tensor of an anisotropic fluid is in the form

$$T_j^i = diag\left[T_0^0,\ T_1^1,\ T_2^2,\ T_3^3\right]. \tag{3}$$

We parametrize it as follows,

$$\begin{aligned} T_j^i &= diag\left[-\rho,\ p_x,\ p_y,\ p_z\right] \\ &= diag\left[-1,\ \omega_x,\ \omega_y,\ \omega_z\right]\rho \\ &= diag\left[-1, \omega, \omega+\gamma, \omega+\gamma\right]\rho\,, \end{aligned} \tag{4}$$

where ω denotes the deviation free EoS parameter of the fluid, $\omega_x, \omega_y, \omega_z$ are the directional EoS parameters of the fluid on x, y and z axes respectively, ρ is the energy density of the fluid. p_x, p_y, p_z are the pressures on x, y and z axes respectively.

Now parametrizing the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameter γ that is the same deviations from ω respectively on y and z axes. ω and γ are not necessarily constants and can be functions of the cosmic time t.

In a co-moving coordinate system, Einstein's field equations (2) for the metric (1) with the help of equation (4) yields,

$$2\frac{\dot a \dot b}{ab} + \frac{1}{b^2} + \frac{\dot b^2}{b^2} - \frac{a^2}{4b^4} = \rho\,. \tag{5}$$

$$2\frac{\ddot b}{b} + \frac{1}{b^2} + \frac{\dot b^2}{b^2} - \frac{3a^2}{4b^4} = -\omega\rho \tag{6}$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{b} + \frac{\dot a \dot b}{ab} + \frac{a^2}{4b^4} = -(\omega+\gamma)\rho \tag{7}$$

The over dot $(\cdot)$ denotes the differentiation with respect to cosmic time t.

3. ISOTROPIZATION AND THE SOLUTIONS :

The directional Hubble parameters in the directions of x, y and z axes for the metric (1) are defined as

$$H_x = \frac{\dot a}{a},\ H_y = H_z = \frac{\dot b}{b}\,. \tag{8}$$

The mean Hubble parameter is

$$H = \frac{1}{3}\frac{\dot V}{V} = \frac{1}{3}\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right), \tag{9}$$

where the spatial volume of the universe is

$$V = ab^2\,. \tag{10}$$

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion.

The anisotropic parameter of the expansion is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{11}$$

where H_i $(i = 1,2,3)$ represent the directional Hubble parameters in the directions of x, y and z respectively. $\Delta = 0$ corresponds to isotropic expansion. The space approaches isotropy, in case of diagonal energy momentum tensor ($T^{0i} = 0$, where $i = 1,2,3$) if $\Delta \to 0$, $V \to \infty$ and $T^{00} > 0$ $(\rho > 0)$ as $t \to \infty$. (Collins and Hawking [10]).

Using $H_y = H_z$ and simplifying the equation (11), we get

$$\Delta = \frac{2}{9H^2}\left(H_x - H_y\right)^2, \tag{12}$$

where $(H_x - H_y)$ is the difference between the expansion rates on x and y axes which can be obtained by using the field equations.

On subtracting (6) from equation (7), we obtain

$$H_x - H_y = \frac{\dot a}{a} - \frac{\dot b}{b} = \frac{\lambda}{V} + \frac{1}{V}\int\left(\frac{b^2 - a^2}{b^4} - \gamma\rho\right)Vdt\,, \tag{13}$$

where λ is a constant of integration and the term with γ is the term that arises due to the possible intrinsic anisotropy of the fluid.

To obtain the anisotropy parameter of the expansion, we use (13) in (12) and obtain

$$\Delta = \frac{2}{9H^2}\left(\lambda + \int\left(\frac{b^2 - a^2}{b^4} - \gamma\rho\right)Vdt\right)^2 V^{-2}. \tag{14}$$

Choosing γ=0, the anisotropy parameter of the expansion for a Bianchi type-IX cosmological model in the presence of perfect fluid reduces to

$$\Delta = \frac{2}{9H^2}\left(\lambda + \int\left(\frac{b^2 - a^2}{b^4}\right)Vdt\right)^2 V^{-2}. \tag{15}$$

The integral term in (14) vanishes for

$$\gamma = \frac{b^2 - a^2}{\rho b^4}. \tag{16}$$

This leads to the following energy momentum tensor

$$T_j^i = diag\left[-1,\ \omega,\ \omega + \frac{b^2 - a^2}{\rho b^4},\ \omega + \frac{b^2 - a^2}{\rho b^4}\right]\rho . \quad (17)$$

Using (16) in equation (14) therein, the anisotropic parameter of the expansion reduced to

$$\Delta = \frac{2}{9}\frac{\lambda^2}{H^2}V^{-2} \quad . \quad (18)$$

It is seen that the above anisotropic parameter of the expansion is exactly same for exponential expansion in Bianchi type-III space time (Akarsu *et. al.* [5]) for anisotropic fluid and is equivalent to ones obtained for exponential expansion in Bianchi type-I (Kumar *et. al.* [14]) and Bianchi type-V (C. P. Singh *et. al.* [20], J. P. Singh *et. al.* [21]) cosmological models with isotropic fluid.

The vanishing of the integral term also reduces the difference between the expansion rates on x and y to the following form

$$H_x - H_y = \frac{\lambda}{V} = \frac{\lambda}{ab^2} \quad . \quad (19)$$

The most general form of the energy density in Bianchi type-IX framework by using the field equation (5) and the definition of the anisotropic parameter of the expansion (11) is obtained as

$$\rho = 3H^2\left(1 - \frac{\Delta}{2}\right) + \frac{4b^2 - a^2}{4b^4} . \quad (20)$$

Using (17) in the Einstein's field equations (5-7) we have

$$\frac{\dot{b}^2}{b^2} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{3a^2}{4b^4} = (1-\gamma)\rho \quad (21)$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{a^2}{2b^4} = -(\omega+\gamma)\rho \quad (22)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{a^2}{4b^4} = -(\omega+\gamma)\rho . \quad (23)$$

To obtain the solution of the field equations, there are three linearly independent equations (21-23) and four unknown functions a, b, ω, ρ. An extra equation is needed to solve the system completely. Therefore we used two different volumetric expansion laws:

$$V = C_1 e^{3kt} \quad (24)$$

and $$V = C_1 t^{3m} , \quad (25)$$

where C_1, k and m are positive constants.

The models with the exponential expansion and power law for $m > 1$ exhibit accelerating volumetric expansion. On the other hand the model for $m = 1$ exhibits volumetric expansion with constant velocity, whereas the model for $m < 1$ exhibits decelerating volumetric expansion. The anisotropic fluid we take here can be considered in the context of DE in the models with exponential expansion and power law expansion for $m > 1$. Riess *et. al.* [17,18] and Perlmutter *et. al.* [15] have shown that the decelerating parameter of the universe is in the range $-1 \le q \le 0$ and the present day universe is undergoing accelerated expansion.

4. MODEL WITH EXPONENTIAL EXPANSION $V=C_1e^{3kt}$:

The scale factors a and b are obtained by solving the field equations (21-23) for the exponential volumetric expansion (24) by considering (19),

$$a = \left(C_1 C_2^2\right)^{1/3} \exp(kt - \frac{2}{9}\frac{\lambda}{kC_1}e^{-3kt}) \quad (26)$$

$$b = \left(\frac{C_1}{C_2}\right)^{1/3} \exp(kt + \frac{1}{9}\frac{\lambda}{kC_1}e^{-3kt}) , \quad (27)$$

where C_2 is a positive constant of integration.

The mean Hubble parameter is

$$H = k \quad (28)$$

and the directional Hubble parameters on the x, y and z axes are respectively,

$$H_x = k + \frac{2}{3}\frac{\lambda}{C_1}e^{-3kt} \quad \text{and}$$

$$H_y = k - \frac{1}{3}\frac{\lambda}{C_1}e^{-3kt} = H_z . \quad (29)$$

From equation (12), the anisotropic parameter of the expansion is obtained by using the directional and mean Hubble parameters,

$$\Delta = \frac{2}{9}\frac{\lambda^2}{k^2 C_1^2}e^{-6kt} . \quad (30)$$

It is observed that the above anisotropic parameter of the expansion is exactly same for exponential expansion in Bianchi type-III space time (Akarsu *et. al.* [5]) for anisotropic fluid and is equivalent to ones obtained for exponential expansion in Bianchi type-I (Kumar *et. al.* [14]) and Bianchi type-V (C. P. Singh *et. al.* [20], J. P. Singh *et. al.* [21]) cosmological models with isotropic fluid.

The energy density of the fluid is obtained from equation (20) by using the scale factor

$$\rho = 3k^2 - \frac{\lambda^2}{3C_1^2}e^{-6kt} + \left(\frac{C_2}{C_1}\right)^{2/3} e^{-2kt - \frac{2}{9}\frac{\lambda}{k}\frac{}{C_1}e^{-3kt}} - \frac{1}{4}\left(\frac{C_2}{C_1}\right)^{2/3} e^{-2kt - \frac{8}{9}\frac{\lambda}{kC_1}e^{-3kt}} . \quad (31)$$

Using (26) and (31) in equation (263), the deviation free part of the anisotropic EoS parameter is obtained as

$$\omega=-\frac{3\left(k-\frac{1}{3}\frac{\lambda}{C_1}e^{-3kt}\right)^2+\frac{2\lambda k}{C_1}e^{-3kt}+\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{2}{9}\frac{\lambda}{kC_1}e^{-3kt}}-\frac{3}{4}C_2^2\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{8}{9}\frac{\lambda}{kC_1}e^{-3kt}}}{3k^2-\frac{\lambda^2}{3C_1^2}e^{-6kt}+\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{2}{9}\frac{\lambda}{kC_1}e^{-3kt}}-\frac{1}{4}C_2^2\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{8}{9}\frac{\lambda}{kC_1}e^{-3kt}}} \tag{32}$$

Also using equations (26) and (27) in equation (16), the skewness parameter γ which is the deviation of ω on *z*-axis is obtained as

$$\gamma=\frac{\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{2}{9}\frac{\lambda}{kC_1}e^{-3kt}}-C_2^2\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{8}{9}\frac{\lambda}{kC_1}e^{-3kt}}}{3k^2-\frac{\lambda^2}{3C_1^2}e^{-6kt}+\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{2}{9}\frac{\lambda}{kC_1}e^{-3kt}}-\frac{1}{4}C_2^2\left(\frac{C_2}{C_1}\right)^{2/3}e^{-2kt-\frac{8}{9}\frac{\lambda}{kC_1}e^{-3kt}}}. \tag{33}$$

From equation (30), we should note that the anisotropy of the expansion (Δ) is not promoted by the anisotropy of the fluid and decreases to null exponentially as t increases. The space approaches to isotropy in this model as $\Delta \to 0$, $V \to \infty$ and $\rho = 3k^2 > 0$ as $t \to \infty$. The energy density (ρ), the deviation free EoS parameter (ω) and the skewness parameter (γ) are dynamical. Also there is no big-bang type of singularity for particular choice of parameters. As $t \to \infty$, the anisotropic fluid isotropizes and mimics the vacuum energy which is mathematically equivalent to the cosmological constant (Λ) i.e. as $t \to \infty$ we get $\gamma \to 0$, $\omega \to -1$ and $\rho \to 3k^2$ as in figure 1, 2 and 3 respectively.

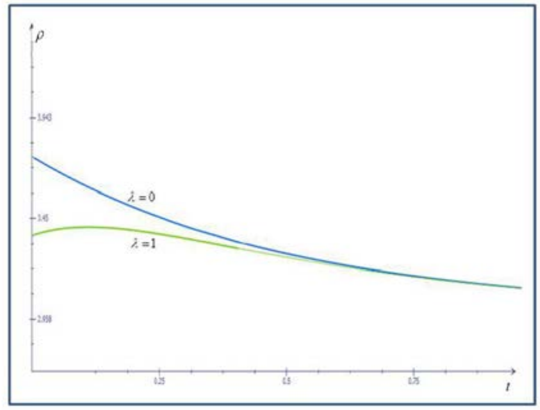

Figure 1. Evolution of Energy Density (ρ) for $k = C_1 = C_2 = 1$, $\lambda = 0$ and $\lambda = 1$

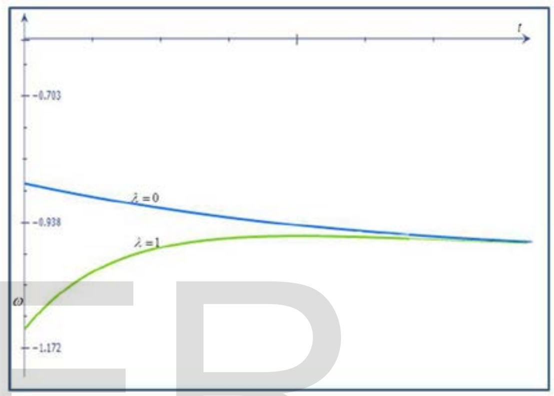

Figure 2. Evolution of Deviation free Parameter (ω) for $k = C_1 = C_2 = 1$, $\lambda = 0$ and $\lambda = 1$

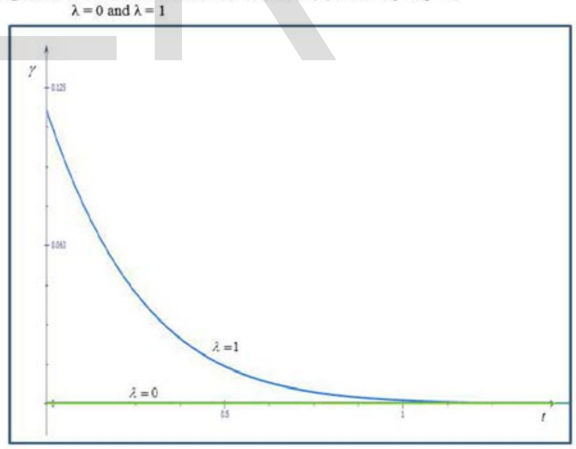

Figure 3. Evolution of Skewness parameter (ϒ) for $k = C_1 = C_2 = 1$, $\lambda = 0$ and $\lambda = 1$

The universe approaches to isotropy monotonically even in the presence of the anisotropic fluid and the anisotropic fluid isotropizes and evolves to the cosmological constant in case of exponential volumetric expansion.

5. MODEL WITH POWER LAW EXPANSION $V = C_1 t^{3m}$:

In power law volumetric expansion, the scale factors a and b are obtained by solving the field equations (21-23) for the exponential volumetric expansion (25) by considering (19),

$$a = \left(C_1 C_2^2\right)^{1/3} t^m e^{\frac{2}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}} \quad (34)$$

$$b = \left(\frac{C_1}{C_2}\right)^{1/3} t^m e^{-\frac{\lambda}{3C_1}\frac{t^{1-3m}}{1-3m}}, \quad (35)$$

where C_2 is a positive constant of integration.

The mean Hubble parameter is

$$H = \frac{m}{t}. \quad (36)$$

The directional Hubble parameters on the x, y and z axes are respectively

$$H_x = \frac{m}{t} + \frac{2}{3}\frac{\lambda}{C_1} t^{-3m} \quad \text{and}$$

$$H_y = \frac{m}{t} - \frac{1}{3}\frac{\lambda}{C_1} t^{-3m} = H_z. \quad (37)$$

From equation (12), the anisotropic parameter of the expansion is obtained by using the directional and mean Hubble parameters as

$$\Delta = \frac{2}{9}\frac{\lambda^2}{C_1^2}\frac{t^{2-6m}}{m^2}. \quad (38)$$

The energy density of the fluid is obtained from equation (20) by using the scale factor

$$\rho = 3m^2 t^{-2} - \frac{\lambda^2}{3C_1^2} t^{-6m} + \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{2}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}} - \frac{1}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{8}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}}. \quad (39)$$

Using (34) and (39) in equation (23), the deviation free part of the anisotropic EoS parameter is obtained as

$$\omega = -\frac{3\left(\frac{m}{t} - \frac{1}{3}\frac{\lambda}{C_1} t^{-3m}\right)^2 - 2mt^{-2} + \frac{2\lambda m}{C_1} t^{-3m-1} + \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{2}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}} - \frac{3}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{8}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}}}{3m^2 t^{-2} - \frac{\lambda^2}{3C_1^2} t^{-6m} + \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{2}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}} - \frac{1}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{8}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}}}. \quad (40)$$

Also using equations (34) and (35) in equation (16), the skewness parameter γ which is the deviation of ω on z-axis is obtained as

$$\gamma = \frac{\left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{2}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}} - C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{8}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}}}{3m^2 t^{-2} - \frac{\lambda^2}{3C_1^2} t^{-6m} + \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{2}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}} - \frac{1}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3} t^{-2m} e^{\frac{8}{3}\frac{\lambda}{C_1}\frac{t^{1-3m}}{1-3m}}} \quad (41)$$

From equation (38), it is observed that, the anisotropy of the expansion (Δ) is not promoted by the anisotropy of the fluid. It behaves monotonically, decays to zero $(\Delta \to 0)$ for $m > \frac{1}{3}$ and diverges $(\Delta \to \infty)$ for $m < \frac{1}{3}$ as $t \to \infty$ and is constant $\left(\Delta \to \frac{2\lambda^2}{9m^2 c_1^2}\right)$ for $m = \frac{1}{3}$.

The spatial volume of the universe expands indefinitely for all values of m i.e. as $t \to \infty, V \to \infty$. The universe approaches to isotropy i.e. $\Delta \to 0$ and $V \to \infty$ as $t \to \infty$ for $m > 1$. Also we get $\omega \to -1$ and $\gamma \to 0$ as $t \to \infty$ indicating that the EoS parameter of the fluid isotropizes and approaches a value in quintessence region with regard to value of m at later times of the universe for accelerating models.

For the model with $m = 1$, we have

$$\omega = \frac{-1 - \left(\frac{C_2}{C_1}\right)^{2/3} - \frac{3}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3}}{3 + \left(\frac{C_2}{C_1}\right)^{2/3} - \frac{1}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3}}$$

and

$$\gamma = \frac{\left(\frac{C_2}{C_1}\right)^{2/3} - C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3}}{3 + \left(\frac{C_2}{C_1}\right)^{2/3} - \frac{1}{4} C_2^2 \left(\frac{C_2}{C_1}\right)^{2/3}}$$

$\Delta \to 0$ and $V \to \infty$ as $t \to \infty$ indicating that, the space approaches to isotropy for the model with $m = 1$ whereas the fluid does not approach to isotropy i.e. $\rho \to 0$ as $t \to \infty$ for $m = 1$ as shown in figure 4. For the model with exponential expansion, the universe approaches to isotropy monotonically even in the presence of anisotropic fluid for $m = 1$ and for $m > 1$ with appropriate values of constant. However, the anisotropic isotropizes only in the accelerating models $(m > 1)$ at later times of the universe and its EoS parameter evolves into the quintessence region. The volume of the universe expands indefinitely for all values of m .

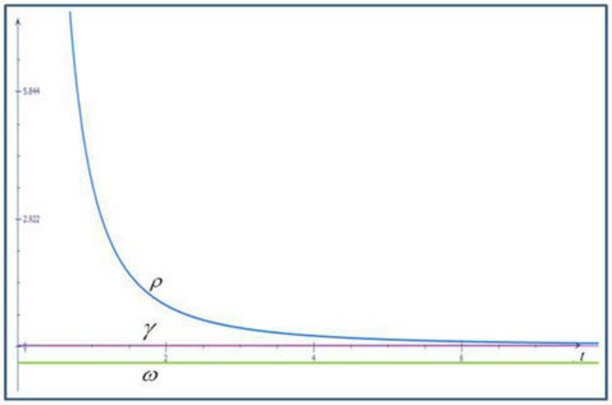

Figure 4. Evolution of Energy Density (ρ), Skewness parameter (ϒ) and Deviation free Parameter (ω) for $m = C_1 = C_2 = 1, \lambda = 0$

6. CONCLUSION:

It is conclude that even in the presence of an anisotropic fluid, Bianchi type-IX universe can approach to isotropy monotonically. The anisotropy of the fluid isotropizes at later times of the universe in the accelerating models. The fluid evolves into the vacuum energy with , $\omega = -1$, which is mathematically equivalent to the cosmological constant (Λ) at the later times of the universe in the model for exponential expansion. Here in this model, we see an isotropic expansion but the possibility of dark energy with an anisotropic equation of state cannot be ruled out. In addition, an anisotropic dark energy does not necessarily distort the symmetry of the space and consequently even if it turns out the spherical symmetry of the universe that achieved during inflation has not distorted in the later times of the universe. It is interesting to note that our investigations resembles with the investigations of Adhav *et. al.* [1, 2, 3, 4].

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